

Rare Radiative $B_c \rightarrow D_{s1}(2460)\gamma$ Transition in QCD

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Abstract

We investigate the radiative $B_c \rightarrow D_{s1}\gamma$ transition in the framework of QCD sum rules. In particular, using the quark condensate, mixed and two-gluon condensate diagrams as well as propagation of the soft quark in the electromagnetic field as non-perturbative effects, we calculate the transition form factors responsible for this decay in both weak annihilation and electromagnetic penguin channels. These form factors then are used to estimate the decay rates and branching ratios in these channels. The total branching ratio for this decay is evaluated to be in the order of 10^{-5} and the dominant mode is the weak annihilation channel.

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I. INTRODUCTION

The B_c is the only heavy meson consists of two heavy quarks with different flavors, hence decay properties of this meson are of special interest. The difference of heavy quark flavors forbids annihilation of this meson into gluons, as a result of which the excited B_c meson states lying below the threshold of decay into the pair of heavy B and D mesons undergo pionic or radiative transitions to the pseudoscalar ground state which is more stable compared to the corresponding quarkonia and decays mostly weakly. It is expected that the experimental studies on the B_c meson and its decay properties constitute one of the important parts of physics program at LHC. The study of heavy mesons will provide not only a window in extracting the most accurate values of Cabbibo- Kobayashi- Maskawa(CKM) matrix elements as the sources of CP -violation in Standard Model (SM) but also in understanding the perturbative and non-perturbative aspects of QCD.

In the present work, we work out the rare radiative $B_c \rightarrow D_{s1}(2460)\gamma$ transition in the framework of QCD sum rules [1, 2]. Here, the $D_{s1}(2460)$ is the axial vector charmed-strange meson with quantum numbers $J^P = 1^+$ and interpolating current $\eta_\nu = \bar{s}\gamma_\nu\gamma_5 c$. This transition proceeds via both weak annihilation (WA) and electromagnetic penguin (EP) of flavor changing neutral current (FCNC) transition, based on $b \rightarrow s\gamma$ at quark level. Using the quark condensate, mixed and two-gluon condensate diagrams as well as propagation of the soft quark in the electromagnetic field as non-perturbative effects, we calculate the transition form factors responsible for this decay in both weak annihilation and electromagnetic penguin modes. We then use these form factors to estimate the decay rates and branching ratios in these modes as well as the total decay rate and branching fraction. As it is expected, the dominant contribution comes from the weak annihilation part. Note that the $B_c \rightarrow D_s^*\gamma$ transition is studied in the same framework in [3]. Some other radiative channels of the B_c meson like $B_c \rightarrow l\bar{\nu}\gamma$ and $B_c \rightarrow B_u^*\gamma$ have also previously studied using QCD sum rules technique [4, 5]. For some other decay channels of the B_c meson see for instance [6–9].

The outline of the paper is as follows. In next section, considering the radiation of the photon from both B_c and D_{s1} mesons, we construct the transition amplitude for the weak annihilation channel in terms of four relevant form factors. Two of the form factors $F_V^{(B_c)}$ and $F_A^{(B_c)}$ responsible for the emission of the photon from the initial state are calculated

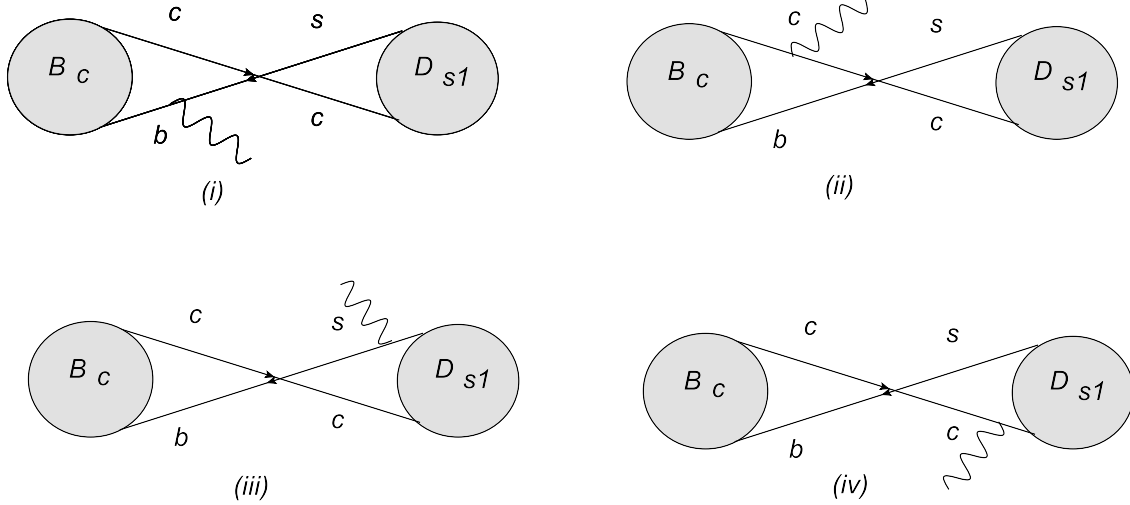


FIG. 1. The weak annihilation mechanism for $B_c \rightarrow D_{s1}\gamma$.

in [4]. The remaining two form factors, $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ representing the emission of the photon from D_{s1} meson are calculated in section III. In section IV, considering the two gluon condensate contributions, we calculate the transition form factors responsible for the electromagnetic penguin mode. Finally, section V is devoted to the numerical analysis of the form factors, calculation of the decay rates and branching ratios for the modes under consideration together with total decay rate and branching ratio. This section also contains our concluding remarks.

II. WEAK ANNIHILATION AMPLITUDE

In this section, we construct the weak annihilation amplitude for the radiative $B_c \rightarrow D_{s1}\gamma$ transition. Considering the quark contents of the initial and final mesonic states, the possible diagrams are shown in figure 1. Taking into account these diagrams, the transition amplitude for the radiative decay under consideration is written as

$$M^{WA}(B_c \rightarrow D_{s1}\gamma) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \langle D_{s1}(p) \gamma(q) | (\bar{s} \Gamma_\nu c) (\bar{c} \Gamma^\nu b) | B_c(p+q) \rangle, \quad (1)$$

where G_F is the Fermi weak coupling constant, V_{ij} are elements of the CKM matrix, $\Gamma_\nu = \gamma_\nu(1-\gamma_5)$; and p , q and $p+q$ are the momenta of the D_{s1} , photon and B_c meson, respectively. To proceed further, we use the factorization hypothesis and write the matrix elements in

Eq.(1) as

$$\langle D_{s1}(p)\gamma(q)|(\bar{s}\Gamma_\nu c)(\bar{c}\Gamma^\nu b)|B_c(p+q)\rangle = -e\varepsilon^\mu\varepsilon^{(D_{s1})\nu}f_{D_{s1}}m_{D_{s1}}T_{\mu\nu}^{(B_c)} - ie\varepsilon^\mu(p+q)^\nu f_{B_c}T_{\mu\nu}^{(D_{s1})}, \quad (2)$$

where we have divided the matrix elements into two separate parts: emission of the photon from B_c meson (diagrams i and ii in figure 1), represented by the covariant decomposition $T_{\mu\nu}^{(B_c)}$ and the emission of the photon from D_{s1} meson denoted by the decomposition $T_{\mu\nu}^{(D_{s1})}$ (see diagrams iii and iv in figure 1). In Eq.(2), the f_{B_c} and $f_{D_{s1}}$ are the decay constants of the B_c and D_{s1} mesons, respectively. The ε^μ and $\varepsilon^{(D_{s1})\nu}$ stand respectively for the polarization vectors of the photon and D_{s1} meson. The covariant decompositions $T_{\mu\nu}^{(B_c)}$ and $T_{\mu\nu}^{(D_{s1})}$ are defined in the following way:

$$T_{\mu\nu}^{(B_c)}(p, q) = i \int d^4x e^{iqx} \langle 0 | T \left\{ j_\mu^{em}(x) \bar{c}(0) \Gamma_\nu b(0) \right\} | B_c(p+q) \rangle \quad (3)$$

and

$$T_{\mu\nu}^{(D_{s1})}(p, q) = i \int d^4x e^{iqx} \langle D_{s1}(p) | T \left\{ j_\mu^{em}(x) \bar{s}(0) \Gamma_\nu c(0) \right\} | 0 \rangle, \quad (4)$$

where j_μ^{em} is the electromagnetic current and T is the time ordering operator. Applying the Ward identity for the electromagnetic current and using $q^2 = 0$ for the real photon as well as $\varepsilon \cdot q = 0$ and $\varepsilon^{(D_{s1})} \cdot p = 0$ similar to what is done in [3] and [10], we get the following results correspond to the emission of the photon from the initial and final mesonic states in terms of form factors:

$$\begin{aligned} e\varepsilon^\mu\varepsilon^{(D_{s1})\nu}f_{D_{s1}}m_{D_{s1}}T_{\mu\nu}^{(B_c)} = ef_{D_{s1}}m_{D_{s1}} \left\{ \left[\left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) (p \cdot q) - (\varepsilon \cdot p) \left(\varepsilon^{(D_{s1})} \cdot q \right) \right] iF_A^{(B_c)} \right. \\ \left. + if_{B_c} \left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) + \varepsilon_{\nu\mu\lambda\sigma} \varepsilon^{(D_{s1})\nu} \varepsilon^\mu p^\lambda q^\sigma F_V^{(B_c)} \right\}, \end{aligned} \quad (5)$$

$$\begin{aligned} ie\varepsilon^\mu(p+q)^\nu f_{B_c}T_{\mu\nu}^{(D_{s1})} = ie f_{B_c} \left\{ \left[\left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) (p \cdot q) - (\varepsilon \cdot p) \left(\varepsilon^{(D_{s1})} \cdot q \right) \right] iF_A^{(D_{s1})} \right. \\ \left. + f_{D_{s1}}m_{D_{s1}} \left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) + \varepsilon_{\nu\mu\lambda\sigma} \varepsilon^{(D_{s1})\nu} \varepsilon^\mu p^\lambda q^\sigma F_V^{(D_{s1})} \right\} \end{aligned} \quad (6)$$

where $F_{V(A)}^{(B_c)}$ and $F_{V(A)}^{(D_{s1})}$ are the transition form factors. Using these expressions, finally, we get the WA transition amplitude as

$$\begin{aligned}
M^{WA}(B_c \rightarrow D_{s1}\gamma) = & e \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left(-f_{D_{s1}} m_{D_{s1}} \left\{ \left[\left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) (p \cdot q) - (\varepsilon \cdot p) \left(\varepsilon^{(D_{s1})} \cdot q \right) \right] i F_A^{(B_c)} + i f_{B_c} \left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) \right. \right. \\
& + \varepsilon_{\nu\mu\lambda\sigma} \varepsilon^{(D_{s1})\nu} \varepsilon^\mu p^\lambda q^\sigma F_V^{(B_c)} \left. \right\} - i f_{B_c} \left\{ \left[\left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) (p \cdot q) - (\varepsilon \cdot p) \left(\varepsilon^{(D_{s1})} \cdot q \right) \right] i F_A^{(D_{s1})} \right. \\
& \left. \left. + f_{D_{s1}} m_{D_{s1}} \left(\varepsilon \cdot \varepsilon^{(D_{s1})} \right) + \varepsilon_{\nu\mu\lambda\sigma} \varepsilon^{(D_{s1})\nu} \varepsilon^\mu p^\lambda q^\sigma F_V^{(D_{s1})} \right\} \right). \tag{7}
\end{aligned}$$

As we previously mentioned, the form factors $F_V^{(B_c)}$ and $F_A^{(B_c)}$ corresponding to the emission of the photon from B_c meson are calculated in [4]. We calculate the remaining two form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ in the next section.

III. QCD SUM RULES FOR THE FORM FACTORS $F_V^{(D_{s1})}$ AND $F_A^{(D_{s1})}$

To calculate the transition form factors in QCD sum rules formalism, we start considering the following correlation function:

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{iQx} \langle \gamma(q) | T \left\{ \bar{c}(x) \gamma_\mu (1 - \gamma_5) s(x) \bar{s}(0) \gamma_\nu \gamma_5 c(0) \right\} | 0 \rangle, \tag{8}$$

where $Q = p + q$. The basic idea in this method is to calculate this correlation function once in hadronic language called phenomenological or physical side and then in terms of QCD degrees of freedom using the operator product expansion in deep Euclidean space, called the theoretical or QCD side. The two representations are then matched in order to get the QCD sum rules for the form factors. To suppress the contributions coming from the higher states and continuum, we apply Borel transformation as well as continuum subtraction. These procedures bring two auxiliary parameters, namely Borel mass parameter as well as continuum threshold that we shall find their working regions such that the physical observables be independent of these parameters in these regions.

First we focus on calculation of the phenomenological side. For this aim, we insert a full set of hadronic D_{s1} state to Eq.(8). After performing integral over d^4x , we get

$$\Pi_{\mu\nu}(p, q) = \frac{\langle \gamma(q) | \bar{c} \gamma_\mu (1 - \gamma_5) s | D_{s1}(p) \rangle \langle D_{s1}(p) | \bar{s} \gamma_\nu \gamma_5 c | 0 \rangle}{m_{D_{s1}}^2 - p^2}. \tag{9}$$

The matrix element, $\langle D_{s1}(p) | \bar{s} \gamma_\nu \gamma_5 c | 0 \rangle$ is defined in terms of decay constant and polarization vector of D_{s1} meson,

$$\langle D_{s1}(p) | \bar{s} \gamma_\nu \gamma_5 c | 0 \rangle = f_{D_{s1}} m_{D_{s1}} \varepsilon_\nu^{(D_{s1})}, \quad (10)$$

while the transition matrix element is parametrized in terms of form factors,

$$\begin{aligned} \langle \gamma(q) | \bar{c} \gamma_\mu (1 - \gamma_5) s | D_{s1}(p) \rangle = e \left\{ i \varepsilon_{\mu\alpha\beta\sigma} \varepsilon^\alpha \varepsilon^{(D_{s1})\beta} q^\sigma \frac{F_V^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2} \right. \\ \left. + \left[\varepsilon_\mu (\varepsilon^{(D_{s1})} \cdot q) - (\varepsilon \cdot \varepsilon^{(D_{s1})}) q_\mu \right] \frac{F_A^{(D_{s1})}(Q^2)}{m_{(D_{s1})}^2} \right\}. \end{aligned} \quad (11)$$

Putting all above equations together and summing over the polarization vector of the D_{s1} meson, we get the following result for the phenomenological part of the correlation function:

$$\Pi_{\mu\nu}(p, q) = \frac{e f_{D_{s1}} m_{D_{s1}}}{m_{D_{s1}}^2 - p^2} \left\{ i \varepsilon_{\mu\nu\alpha\sigma} \varepsilon^\alpha q^\sigma \frac{F_V^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2} + \left[q_\mu \varepsilon_\nu - \varepsilon_\mu q_\nu \right] \frac{F_A^{(D_{s1})}(Q^2)}{m_{D_{s1}}^2} \right\}. \quad (12)$$

The QCD side of the correlation function is calculated in deep Euclidean region in terms of QCD parameters. To proceed, first we write the correlation function in terms of two selected structures as

$$\Pi_{\mu\nu}(p, q) = i \varepsilon_{\mu\nu\alpha\sigma} \varepsilon^\alpha q^\sigma \Pi_1 + \left[q_\mu \varepsilon_\nu - \varepsilon_\mu q_\nu \right] \Pi_2, \quad (13)$$

where each function Π_i ($i = 1$ or 2) has perturbative and non-perturbative parts, i.e.,

$$\Pi_i = \Pi_i^{pert} + \Pi_i^{non-pert}. \quad (14)$$

For calculation of the perturbative parts, we consider (a) and (b) in figure 2 where photon can be radiated from both charm and strange quarks. For the non-perturbative effects, we take into account the quark condensate and mixed diagrams [Figs. 2(c), 2(d) and 2(e)] as well as diagram 2(f) indicating the interaction of the photon with the soft quark.

The perturbative part in each case can be written via the dispersion relation as

$$\Pi_i^{pert} = \int ds \frac{\rho_i(s, p^2)}{s - Q^2} + \text{subtraction terms}. \quad (15)$$

where ρ_i are the spectral densities. Our main task in this section is to calculate these spectral densities using the diagrams (a) and (b) in figure 2. Here we use a Feynman and Shwinger parameterization based method with several Borel transformations (see also [11]).

The Feynman amplitude for the diagram (a) can be written as

$$\Pi_{\mu\nu,(a)} = e N_c Q_s \int \frac{d^4 k}{2\pi^4} \left\{ Tr \left[\frac{i(\not{k} + m_c)}{k^2 + m_c^2} \gamma_\mu (1 - \gamma_5) \frac{i(Q + \not{k} + m_s)}{(Q + k)^2 - m_s^2} \not{\epsilon} \frac{i(\not{p} + \not{k} + m_s)}{(p + k)^2 - m_s^2} \gamma_\nu \gamma_5 \right] \right\}. \quad (16)$$

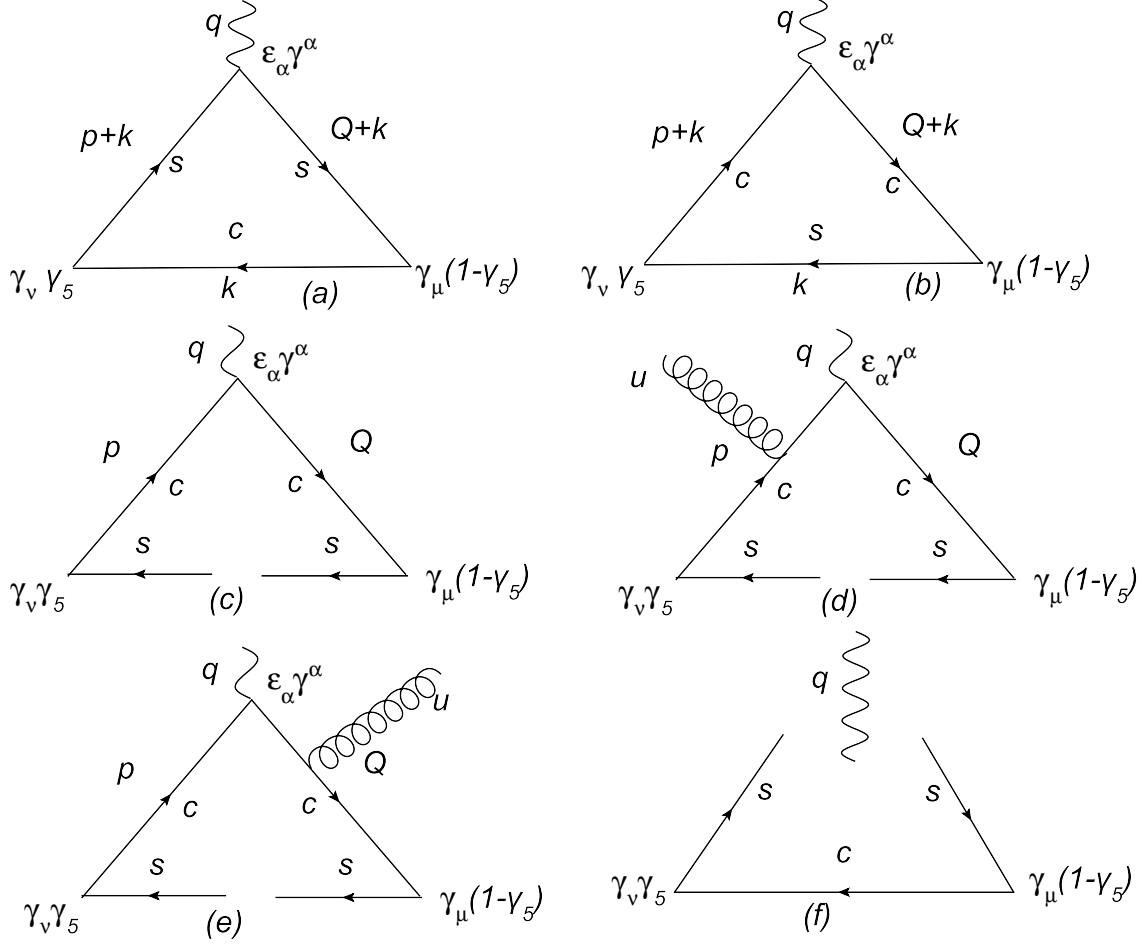


FIG. 2. Diagrams for bare-loop [(a), (b)], power corrections from the operators with 3, 4 and 5 dimensions [(c), (d), (e)] and propagation of the soft quark in electromagnetic field (f).

Using first the Feynman parameterization, we perform integral over d^4k . Then, we use the Shwinger parameterization as

$$\frac{1}{\Delta^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\alpha \alpha^{n-1} e^{-\alpha \Delta}, \quad (17)$$

to write the denominators of the obtained results in exponential forms. As a result, we get

$$\Pi_{1,a}^{pert} = \frac{eN_c Q_s}{4\pi^2} \left\{ \int_0^1 dx x \int_0^1 dy \left[m_s(m_c + m_s xy) + p^2 x \bar{x} (1 - \bar{x} y) + 2p \cdot q \bar{x} x^2 y^2 \right] \int_0^\infty d\alpha e^{-\alpha \Delta} \right\}, \quad (18)$$

$$\Pi_{2,a}^{pert} = \frac{eN_c Q_s}{4\pi^2} \left\{ \int_0^1 dx x \int_0^1 dy x \left[m_s(m_c - m_s xy) - p^2 x \bar{x} (1 - \bar{x} y) - 2p \cdot q \bar{x} x^2 y^2 \right] \int_0^\infty d\alpha e^{-\alpha \Delta} \right\}, \quad (19)$$

where $\bar{x}(\bar{y}) = 1 - x(y)$, and $\Delta = m_c^2 \bar{x} + m_s^2 x - p^2 x \bar{x} \bar{y} - Q^2 x \bar{x} y$.

Applying double Borel transformation $\widehat{B}(M_1^2)\widehat{B}(M_2^2)$ on Π_i^{pert} , that transforms $Q^2 \rightarrow M_1^2$ and $p^2 \rightarrow M_2^2$, we obtain

$$\begin{aligned} \widehat{\Pi}_{1,a}^{pert} = & \frac{eN_c Q_s}{4\pi^2} \frac{\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} \int_0^1 dx \frac{1}{\bar{x}} e^{\frac{(m_c^2 \bar{x} + m_s^2 x)(\sigma_1 + \sigma_2)}{\bar{x}x}} \\ & \times \left\{ m_c m_s + m_s^2 x \frac{\sigma_1}{\sigma_1 + \sigma_2} + p^2 \bar{x} x \left(x \frac{\sigma_1}{\sigma_1 + \sigma_2} + \frac{\sigma_2}{\sigma_1 + \sigma_2} \right) + 2p \cdot q \bar{x} x^2 \frac{\sigma_1^2}{(\sigma_1 + \sigma_2)^2} \right\}, \end{aligned} \quad (20)$$

$$\begin{aligned} \widehat{\Pi}_{2,a}^{pert} = & \frac{eN_c Q_s}{4\pi^2} \frac{\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} \int_0^1 dx \frac{1}{\bar{x}} e^{\frac{(m_c^2 \bar{x} + m_s^2 x)(\sigma_1 + \sigma_2)}{\bar{x}x}} \\ & \times \left\{ m_c m_s - m_s^2 x \frac{\sigma_1}{\sigma_1 + \sigma_2} - p^2 \bar{x} x \left(x \frac{\sigma_1}{\sigma_1 + \sigma_2} + \frac{\sigma_2}{\sigma_1 + \sigma_2} \right) - 2p \cdot q \bar{x} x^2 \frac{\sigma_1^2}{(\sigma_1 + \sigma_2)^2} \right\}, \end{aligned} \quad (21)$$

where $\sigma_{1,2} = 1/M_{1,2}^2$ and we have used the Borel transformation of exponential function as

$$B_{p^2}(M^2)e^{-\alpha p^2} = \delta(1 - \alpha M^2). \quad (22)$$

Now, we perform the second double Borel transformation on $\widehat{\Pi}_i^{pert}$ in order to transform σ_1 and σ_2 to new variables s and t , using

$$\varrho_i(s, t, p^2) = \frac{1}{st} \widehat{B}\left(\frac{1}{s}, \sigma_1\right) \widehat{B}\left(\frac{1}{t}, \sigma_2\right) \frac{\widehat{\Pi}_i^{pert}}{\sigma_1 \sigma_2}, \quad (23)$$

where

$$\widehat{B}\left(\frac{1}{s}, \sigma_1\right) \widehat{B}\left(\frac{1}{t}, \sigma_2\right) e^{-\alpha(\sigma_1 + \sigma_2)} = \delta\left(1 - \frac{\alpha}{s}\right) \delta\left(1 - \frac{\alpha}{t}\right), \quad (24)$$

and we have also used

$$\sigma^n e^{-\alpha \sigma} = \left(-\frac{d}{d\alpha}\right)^n e^{-\alpha \sigma}. \quad (25)$$

The final expressions for the spectral densities are calculated via the following formula:

$$\rho_i(s, p^2) = \int dt \frac{\varrho_i(s, t, p^2)}{t - p^2}. \quad (26)$$

After lengthy calculations, we get the following spectral densities corresponding to the dia-

gram (a):

$$\begin{aligned} \rho_{1a}(s, p^2) = & \frac{eN_c Q_s}{4\pi^2} \frac{1}{(s-p^2)^3} \int_{x_0}^{x_1} dx \frac{1}{\bar{x}^2} \left\{ m_c m_s (s-p^2)^2 \bar{x} - 2m_c^4 (m_s^2 - p^2) \bar{x}^2 \right. \\ & + m_c^2 \bar{x} \left[4m_s^4 x + p^2 (4p^2 + (s-p^2)) \bar{x} x + m_s^2 ((s-p^2) + 4p^2 x^2) \right] \\ & + x \left[-2m_s^6 x + p^2 \bar{x}^2 (s^2 + p^2 s (\bar{x} - 1) + p^4 (\bar{x} + 2)) \right. \\ & \left. \left. - m_s^2 p^2 (-1 + x^2) ((s-p^2) + 2p^2 x) \right] + m_s^4 ((s-p^2) + p^2 (4x^2 - 2x)) \right\}, \end{aligned} \quad (27)$$

$$\begin{aligned} \rho_{2a}(s, p^2) = & \frac{eN_c Q_s}{4\pi^2} \frac{1}{(s-p^2)^3} \int_{x_0}^{x_1} dx \frac{1}{\bar{x}^2} \left\{ m_c m_s (s-p^2)^2 \bar{x} + 2m_c^4 (m_s^2 - p^2) \bar{x}^2 \right. \\ & + x \left[2m_s^6 x - p^2 \bar{x}^2 (s^2 + p^2 s (x - 2) + p^4 (x + 1)) \right. \\ & + m_s^2 p^2 (-1 + x^2) ((s-p^2) + 2p^2 x) - m_s^4 ((s-p^2) + p^2 (4x^2 - 2x)) \left. \right] \\ & \left. - m_c^2 \bar{x} \left[-4m_s^4 x + p^2 (3p^2 + s) \bar{x} x + m_s^2 ((s-p^2) - 4p^2 x^2) \right] \right\}, \end{aligned} \quad (28)$$

where the integral boundaries x_0 and x_1 satisfy the the following inequality:

$$sx\bar{x} - (m_c^2 \bar{x} + m_s^2 x) \geq 0, \quad (29)$$

that comes from the Heaviside-Theta function arising in calculations. Similarly, we calculate the contribution of diagram 2 (b). The final expressions for the spectral densities correspond to the two selected structures are obtained as

$$\begin{aligned} \rho_1(s, p^2) = & \frac{eN_c}{24\pi^2} \frac{1}{(p^2 - s)^3} \left\{ Q_s \left[\lambda \left(6s^3 \alpha (1 - 2\alpha - 2(\alpha - \beta)^2) + 3p^2 s^2 \{ 1 - 4\alpha^3 - 2\alpha(3 + \beta)(1 + 2\beta) \right. \right. \right. \\ & + \beta(2 + 5\beta) + \alpha[2 + \alpha(17 + 8\beta)] \} + p^6(5 + 2\alpha^2 + \alpha(5 - 4\beta) - 7\beta + 2\beta^2) \\ & - p^4 s[4 + 4\alpha^3 - 5\beta - 11\beta^2 - \alpha^2(19 + 8\beta) + \alpha(23 + 26\beta + 4\beta^2)] \left. \right) - 6 \left(m_c m_s (p^2 - s)^2 \right. \\ & + s\alpha[2p^4 + s^2(-1 + 4\alpha)(\alpha - \beta) + p^2 s(2 - 7\alpha + 3\beta)] \left. \right) \ln \left(\frac{1 + \alpha - \beta - \lambda}{1 + \alpha - \beta + \lambda} \right) \left. \right] \\ & + Q_c \left[\lambda \left(6s^3 (1 - 2(\alpha - \beta)^2 - 2\beta)\beta + 3p^2 s^2 (1 + \alpha(2 + 5\alpha) - 2(3 + \alpha)(1 + 2\alpha)\beta - 4\beta^3 \right. \right. \\ & + \beta(2 + (17 + 8\alpha)\beta)) + p^6(5 + 2\beta^2 + \beta(5 - 4\alpha) - 7\alpha + 2\alpha^2) - p^4 s[4 + 4\beta^3 \\ & - 5\alpha - 11\alpha^2 - \beta^2(19 + 8\alpha) + \beta(23 + 26\alpha + 4\alpha^2)] \left. \right) - 6 \left(m_c m_s (p^2 - s)^2 \right. \\ & + s\beta[2p^4 + s^2(-\alpha + \beta)(-1 + 4\beta) + p^2 s(2 + 3\alpha - 7\beta)] \left. \right) \ln \left(\frac{1 - \alpha + \beta - \lambda}{1 - \alpha + \beta + \lambda} \right) \left. \right\}, \end{aligned} \quad (30)$$

and

$$\begin{aligned}
\rho_2(s, p^2) = & \frac{eN_c}{24\pi^2} \frac{1}{(p^2 - s)^3} \left\{ Q_s \left[\lambda \left(p^4 s (4 + \alpha(23 + \alpha(-19 + 4\alpha))) - 5\beta + 2(13 - 4\alpha)\alpha\beta \right. \right. \right. \\
& + (-11 + 4\alpha)\beta^2) + 3p^2 s^2 (-1 + (-4 + \alpha)\alpha(-1 + 4\alpha) - 2\beta + 2(7 - 4\alpha)\alpha\beta \\
& + (-5 + 4\alpha)\beta^2) - p^6 (5 + 2\alpha^2 + \beta(5 + 2\beta) - \alpha(7 + 4\beta)) \\
& + 6s^3 \alpha [-1 + 2(\alpha + \alpha^2 - 2\alpha\beta + \beta^2)] \Big) - 6 \left(m_c m_s (p^2 - s)^2 \right. \\
& - s\alpha [2p^4 + s^2(-1 + 4\alpha)(\alpha - \beta) + p^2 s (2 - 7\alpha + 3\beta)] \Big) \ln \left(\frac{1 + \alpha - \beta - \lambda}{1 + \alpha - \beta + \lambda} \right) \Big] \\
& + Q_c \left[\lambda \left(6s^3 \beta (-1 + 2(\alpha - \beta)^2 + 2\beta) - p^6 (5 + \alpha(5 + 2\alpha) - 7\beta - 4\alpha\beta + 2\beta^2) \right. \right. \\
& + p^4 s \{ 4 + \alpha^2(-11 + 4\beta) + \alpha(-5 + 26\beta - 8\beta^2) + \beta[23 + \beta(-19 + 4\beta)] \} \\
& + 3p^2 s^2 [-1 + \alpha^2(-5 + 4\beta) + (-4 + \beta)\beta(-1 + 4\beta) - 2\alpha(1 - 7\beta + 4\beta^2)] \Big) \\
& - 6 \left(m_c m_s (p^2 - s)^2 + s\beta [-2p^4 + s^2(\alpha - \beta)(-1 + 4\beta) \right. \\
& \left. \left. + p^2 s (-2 - 3\alpha + 7\beta)] \right) \ln \left(\frac{1 - \alpha + \beta - \lambda}{1 - \alpha + \beta + \lambda} \right) \right] \Big\}, \tag{31}
\end{aligned}$$

where $\alpha = \frac{m_s^2}{s}$, $\beta = \frac{m_c^2}{s}$ and $\lambda = \sqrt{1 + \alpha^2 + \beta^2 - 2\alpha - 2\beta - 2\alpha\beta}$.

For the non-perturbative part, we first calculate contributions of the quark condensate and mixed diagrams (diagrams c,d and e in figure 2). After lengthy calculations, we obtain the result as follows:

$$\begin{aligned}
\Pi_{1(c,d,e)}^{non-pert} = & \frac{m_c}{r^2 R^2} \langle \bar{s}s \rangle + \frac{m_s}{2} \langle \bar{s}s \rangle \left[\frac{2}{r^2 R^2} + \frac{m_c^2}{r^4 R^2} - \frac{7m_c^2}{r^2 R^4} - \frac{4m_c^4}{r^4 R^4} \right] \\
& + \frac{m_s^2}{2} \langle \bar{s}s \rangle \left[\frac{2m_c^3}{r^2 R^6} - \frac{8m_c^5}{r^6 R^4} + \frac{2m_c^3}{r^4 R^4} - \frac{3m_c}{r^2 R^4} + \frac{2m_c^3}{r^6 R^2} - \frac{m_c}{r^4 R^2} \right] \\
& + \frac{m_0^2}{12} \langle \bar{s}s \rangle \left[\frac{-6m_c^3}{r^2 R^6} + \frac{24m_c^5}{r^6 R^4} - \frac{6m_c^3}{r^4 R^4} + \frac{8m_c}{r^2 R^4} - \frac{6m_c^3}{r^6 R^2} + \frac{3m_c}{r^4 R^2} \right] - \frac{4m_c^3}{r^2 R^4} \langle \bar{s}s \rangle, \tag{32}
\end{aligned}$$

and

$$\begin{aligned}
\Pi_{2(c,d,e)}^{non-pert} = & -\frac{m_c}{r^2 R^2} \langle \bar{s}s \rangle + \frac{m_s}{2} \langle \bar{s}s \rangle \left[\frac{1}{r^2 R^2} + \frac{1}{R^4} - \frac{4m_c^4}{r^4 R^4} - \frac{3m_c^2}{r^2 R^4} + \frac{m_c^2}{2r^4 R^2} \right] \\
& + \frac{m_s^2}{2} \langle \bar{s}s \rangle \left[-\frac{2m_c^3}{r^2 R^6} + \frac{8m_c^5}{r^6 R^4} - \frac{2m_c^3}{r^4 R^4} + \frac{3m_c}{r^2 R^4} - \frac{2m_c^3}{r^6 R^2} + \frac{m_c}{r^4 R^2} \right] \\
& + \frac{m_0^2}{4} \langle \bar{s}s \rangle \left[\frac{2m_c^3}{r^2 R^6} - \frac{8m_c^5}{r^6 R^4} + \frac{2m_c^3}{r^4 R^4} - \frac{4m_c}{r^2 R^4} + \frac{2m_c^3}{r^6 R^2} - \frac{m_c}{r^4 R^2} \right] + \frac{4m_c^3}{r^2 R^4} \langle \bar{s}s \rangle, \tag{33}
\end{aligned}$$

where $r^2 = p^2 - m_c^2$ and $R^2 = Q^2 - m_c^2$.

The final contribution to the WA mode is that of diagram (f). This diagram corresponds to the propagation of the soft quark in the external electromagnetic field. Here we need to use the light-cone version of the QCD sum rules and photon distribution amplitudes (DAs). The relevant correlation function is of the form:

$$\Pi_{\mu\nu,(f)}(p, q) = i \int d^4x e^{-iQx} \langle \gamma(q) | T \left\{ \bar{s}(0) \gamma_\mu \gamma_5 c(0) \bar{c}(x) \gamma_\nu (1 - \gamma_5) s(x) \right\} | 0 \rangle. \quad (34)$$

Contracting c -quark lines in Eq.(49) and using the propagator of the heavy quark in momentum space, we obtain:

$$\Pi_{\mu\nu,(f)}(p, q) = i^2 \int d^4x \frac{d^4k}{(2\pi)^4} \frac{e^{-i(Q-k)x}}{m_c^2 - k^2} \langle \gamma(q) | \bar{s} \gamma_\mu \gamma_5 (\not{k} + m_c) \gamma_\nu (1 - \gamma_5) s | 0 \rangle. \quad (35)$$

To relate the matrix element exists in the above equation to the photon DAs, we use the identities,

$$\begin{aligned} \gamma_\mu \gamma_\nu &= g_{\mu\nu} + i\sigma_{\mu\nu} \\ \gamma_\mu \gamma_\nu \gamma_5 &= g_{\mu\nu} \gamma_5 - \frac{i}{2} \varepsilon_{\mu\nu\alpha\beta} \sigma_{\alpha\beta} \\ \gamma_\mu \gamma_\alpha \gamma_\nu &= g_{\mu\alpha} \gamma_\nu + g_{\nu\alpha} \gamma_\mu - g_{\mu\nu} \gamma_\alpha + i\varepsilon_{\mu\nu\alpha\lambda} \gamma_\lambda \gamma_5. \end{aligned} \quad (36)$$

The relevant photon DAs of twist 2, 3, and 4 [12, 13] are as follows:

$$\begin{aligned} \langle \gamma(q) | \bar{s} \gamma_\nu s | 0 \rangle &= -\frac{Q_s}{2} f_{3\gamma} \int_0^1 du \bar{\psi}^{(V)}(u) x^\theta F_{\theta\nu}(ux) \\ \langle \gamma(q) | \bar{s} \gamma_\alpha \gamma_5 s | 0 \rangle &= -\frac{iQ_s}{4} f_{3\gamma} \int_0^1 du \bar{\psi}^{(A)}(u) x^\theta \tilde{F}_{\theta\alpha}(ux) \\ \langle \gamma(q) | \bar{s} \sigma_{\alpha\beta} s | 0 \rangle &= Q_s \langle \bar{s}s \rangle \int_0^1 du \phi(u) F_{\alpha\beta}(ux) \\ &\quad + \frac{Q_s \langle \bar{s}s \rangle}{16} \int_0^1 du x^2 \mathbf{A}(u) F_{\alpha\beta}(ux) \\ &\quad + \frac{Q_s \langle \bar{s}s \rangle}{8} \int_0^1 du \mathbf{B}(u) x^\rho (x_\beta F_{\alpha\rho}(ux) - x_\alpha F_{\beta\rho}), \end{aligned} \quad (37)$$

where $F_{\mu\nu}$ is the field strength tensor of the electromagnetic field and is defined by

$$\begin{aligned} F_{\mu\nu}(x) &= -i(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) e^{iqx} \\ \tilde{F}_{\mu\nu}(x) &= \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}(x). \end{aligned} \quad (38)$$

The wave function $\phi(u)$ is defined in terms of the magnetic susceptibility, $\chi(\mu)$ at a renormalization scale ($\mu = 1\text{GeV}^2$) in the following manner:

$$\phi(u) = \chi(\mu) u(1-u). \quad (39)$$

The remaining functions $\bar{\psi}^{(V)}(u)$, $\bar{\psi}^{(A)}(u)$, $\mathbf{A}(u)$, and $\mathbf{B}(u)$ are also defined as [12, 13]:

$$\begin{aligned}
\bar{\psi}^{(V)}(u) &= -20u(1-u)(2u-1) + \frac{15}{16}(\omega_\gamma^A - 3\omega_\gamma^V)u(1-u)(2u-1)(7(2u-1)^2 - 3), \\
\bar{\psi}^{(A)}(u) &= (1-(2u-1)^2)(5(2u-1)^2 - 1)\frac{5}{2}\left(1 + \frac{19}{16}\omega_\gamma^V - \frac{3}{16}\omega_\gamma^A\right), \\
\mathbf{A}(u) &= 40u(1-u)(3k - k^+ + 1) + 8(\xi_2^+ - 3\xi_2) \\
&\quad \times [u(1-u)(2 + 13u(1-u)) + 2u^3(10 - 15u + 16u^2) \ln u \\
&\quad + 2(1-u)^3(10 - 15(1-u) + 6(1-u^2)) \ln(1-u)], \\
\mathbf{B}(u) &= 40 \int_0^u d\alpha(4-\alpha)(1+3k^+)\left[-\frac{1}{2} + \frac{3}{2}(2\alpha-1)^2\right],
\end{aligned} \tag{40}$$

where k , k^+ , ξ_2 , ξ_2^+ and $f_{3\gamma}$ are constants (see [12, 13]). Putting the above equations all together after performing the integrals over d^4x and d^4k , the coefficients of the corresponding structures $i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\alpha q^\beta$ and $[q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]$ are obtained as follows:

$$\begin{aligned}
\Pi_{1f}^{non-pert}(p, q) &= \frac{Q_s}{2(m_c^2 - p^2)^3} \int_0^1 du \left\{ m_c^3 \langle \bar{s}s \rangle \mathbf{A}(u) \right. \\
&\quad \left. + (m_c^2 - p^2) \left[m_c \langle \bar{s}s \rangle \mathbf{B}(u) - 2(5m_c^2 - p^2)(m_c \langle \bar{s}s \rangle \phi(u) - f_{3\gamma} \psi^{(V)}(u)) \right] \right\}, \\
\Pi_{2f}^{non-pert}(p, q) &= \frac{m_c Q_s}{2(m_c^2 - p^2)^3} \int_0^1 du \left\{ \mathbf{A}(u) m_c^2 \langle \bar{s}s \rangle + 2(-5m_c^2 + p^2) \langle \bar{s}s \rangle \phi(u) \right. \\
&\quad \left. + (m_c^2 - p^2) \left[\mathbf{B}(u) \langle \bar{s}s \rangle + f_{3\gamma} m_c \psi^{(A)}(u) \right] \right\}.
\end{aligned} \tag{41}$$

Now, To find the QCD sum rules for the form factors, we match the coefficient of the selected structures from both phenomenological and QCD sides and perform the Borel transformation with respect to the momentum of D_{s1} meson, ($p^2 \rightarrow M_B^2$). To further suppress the contribution of the higher states and continuum, we also perform the continuum subtraction and use the quark-hadron duality assumption. As a result, we get

$$F_{V,A}^{(D_{s1})}(Q^2) = \frac{m_{D_{s1}}}{f_{D_{s1}}} e^{m_{D_{s1}}/M_B^2} \widehat{B} \left\{ \int_{(m_s+m_c)^2}^{s_0} ds \frac{\rho_{1,2}(s, p^2)}{s - Q^2} + \Pi_{1,2(c+d+e+f)}^{non-pert} \right\}, \tag{42}$$

where s_0 is the continuum threshold; and V and A correspond to 1 and 2 in the right hand side, respectively. To obtain the expression of the above sum rules in Borel scheme, we perform the Borel transformation using the following standard rule:

$$\widehat{B} \frac{1}{(p^2 - s)^n} = (-1)^n \frac{e^{-s/M_B^2}}{\Gamma(n)(M_B^2)^{n-1}}. \tag{43}$$

IV. QCD SUM RULES FOR THE FORM FACTORS RESPONSIBLE FOR THE ELECTROMAGNETIC PENGUIN MODE

At quark level, the FCNC based EP transition of $B_c \rightarrow D_{s1}\gamma$ proceeds via $b \rightarrow s\gamma$, whose effective Hamiltonian is written as

$$H^{eff} = -\frac{G_F e}{4\pi^2 \sqrt{2}} V_{tb} V_{ts}^* C_7(\mu) \bar{s} \sigma_{\mu\nu} \left[m_b \frac{1 + \gamma_5}{2} + m_s \frac{1 - \gamma_5}{2} \right] b F^{\mu\nu}. \quad (44)$$

The amplitude of this mode is obtained from

$$M^{EP} = \langle D_{s1}(p) | H^{eff} | B_c(Q) \rangle, \quad (45)$$

hence to proceed further, we need to calculate the following matrix elements:

$$\langle D_{s1} | \bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu b | B_c \rangle, \quad (46)$$

which can be parametrized in terms of two gauge invariant form factors $T_1(q^2)$ and $T_2(q^2)$ in the case of real photon, i.e.

$$\begin{aligned} \langle D_{s1}(p, \varepsilon^{(D_{s1})}) | \bar{s} \sigma_{\mu\nu} q^\nu \gamma_5 b | B_c(Q) \rangle &= i \varepsilon_{\mu\alpha\beta\lambda} \varepsilon^{(D_{s1})\alpha} p^\beta Q^\lambda T_1(0), \\ \langle D_{s1}(p, \varepsilon^{(D_{s1})}) | \bar{s} \sigma_{\mu\nu} q^\nu b | B_c(Q) \rangle &= \left[(m_{B_c}^2 - m_{D_{s1}}^2) \varepsilon_\mu^{(D_{s1})} - (\varepsilon^{(D_{s1})} \cdot q)(p + Q)_\mu \right] T_2(0). \end{aligned} \quad (47)$$

These two form factors are not independent of each other and using the relation $\sigma_{\mu\nu} \gamma_5 = -\frac{i}{2} \varepsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta}$ we read $T_1(0) = \frac{1}{2} T_2(0)$. Therefor, we need to calculate just one of them and in the following, we calculate the $T_2(0)$. The corresponding correlation function is chosen as

$$\Pi_{\mu\nu}(p^2, Q^2) = i^2 \int \int d^4x d^4y e^{-i(Qx - py)} \langle 0 | T \left\{ \bar{c}(y) \gamma_\nu \gamma_5 s(y) \bar{s}(0) \sigma_{\mu\alpha} q^\alpha b(0) \bar{b}(x) \gamma_5 c(x) \right\} | 0 \rangle, \quad (48)$$

where, $\bar{b} \gamma_5 c$ and $\bar{c} \gamma_\nu \gamma_5 s$ are the interpolating currents of the initial and final mesonic states, respectively; and $\bar{s} \sigma_{\mu\alpha} q^\alpha b$ is the transition current. From the general philosophy of the QCD sum rules, we calculate this correlation function again in two different languages, namely hadronic and quark-gluon. For the hadronic or phenomenological part, we get

$$\begin{aligned} \Pi_{\mu\nu}(p^2, Q^2) &= \frac{i f_{D_{s1}} f_{B_c} m_{D_{s1}} m_{B_c}^2}{(m_{B_c}^2 - Q^2)(m_{D_{s1}}^2 - p^2)(m_b + m_c)} \left\{ (m_{B_c}^2 - m_{D_{s1}}^2) g_{\mu\nu} T_2(0) \right. \\ &\quad \left. - \left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{D_{s1}}^2} \right) p_\mu p_\nu T_2(0) + (p + Q)_\mu \left[\frac{p \cdot q}{m_{D_{s1}}^2} p_\nu - q_\nu \right] T_2(0) \right\} + \dots, \end{aligned} \quad (49)$$

where ... denotes contribution of the higher states and continuum which will be suppressed then applying Borel transformation as well as continuum subtraction. To calculate the $T_2(0)$, we choose the structure $g_{\mu\nu}$. In deriving the above equation, we have used the following definition of the decay constant of the B_c meson:

$$\langle B_c | \bar{b} \gamma_5 c | 0 \rangle = i \frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)}. \quad (50)$$

In QCD side, the correlation function is written in terms of the selected structure as

$$\Pi_{\mu\nu} = g_{\mu\nu} \Pi(p^2, Q^2), \quad (51)$$

where

$$\Pi(p^2, Q^2) = \Pi^{pert}(p^2, Q^2) + \Pi^{non-pert}(p^2, Q^2). \quad (52)$$

Here the perturbative part is related to the spectral density, ρ^{pert} by the double dispersion integral,

$$\Pi^{pert}(p^2, Q^2) = -\frac{1}{(2\pi)^2} \int \int ds' dt \frac{\rho^{pert}(p^2, Q^2)}{(s' - Q^2)(t - p^2)} + \text{subtraction terms}, \quad (53)$$

and for the non-perturbative contributions, we will calculate the two-gluon condensate diagrams.

Now, we focus our attention to calculate the spectral density. Using Cutkosky method to calculate the spectral density, we get:

$$\begin{aligned} \rho^{per}(s', t) = 2Nc \Big\{ I_0 \Big[\Delta[(m_b - m_c)(m_c + m_s) + t] - \Delta'[(m_b - m_c)(m_c + m_s) + s'] \\ + 2m_c[(m_c + m_s)s' + m_b t - m_c t] - m_c(m_b + m_s)u \Big] + 2A1(-2s' + u) \Big\}, \end{aligned} \quad (54)$$

where

$$\begin{aligned} I_0 &= \frac{1}{4\sqrt{\lambda'(s', t, q^2)}}, \\ \lambda'(a, b, c) &= a^2 + b^2 + c^2 - 2ab - 2ac - 2bc, \\ A_1 &= \frac{-I_0}{(-4s't + u^2)^2} [\Delta^2 t + \Delta'^2 s' - \Delta \Delta' u + m_c^2(-4s't + u^2)], \\ \Delta &= s' + m_c^2 - m_b^2, \\ \Delta' &= t + m_c^2 - m_s^2, \\ u &= t + s' - q^2. \end{aligned} \quad (55)$$

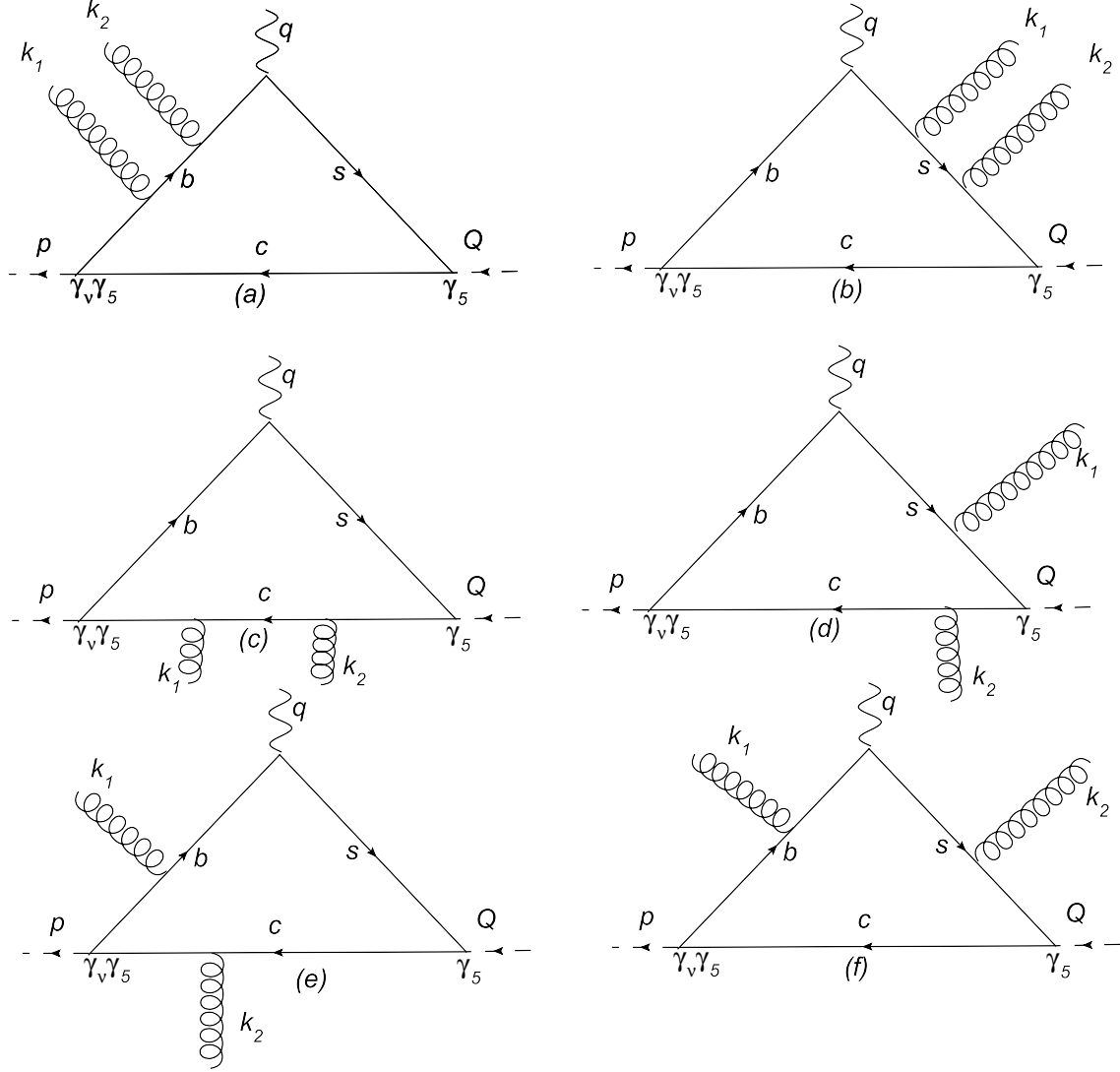


FIG. 3. Feynman diagrams for gluon condensates corrections.

The following inequalities, that comes from the δ -integration of the Cutkosky method, restrict the boundaries of the s' - and t -integrals:

$$\begin{aligned} m_c^2 &\leq t \leq t_0, \\ t - \frac{tm_b^2}{m_c^2 - t} &\leq s' \leq s'_0. \end{aligned} \quad (56)$$

There are several sources for non-perturbative contributions, such as quark, quark-gluon, and gluon condensates. But the quark condensates (s -quark) and quark-gluon condensates give zero contributions after applying the double Borel transformation with respect to Q^2 ($Q^2 \rightarrow M_1^2$) and p^2 ($p^2 \rightarrow M_2^2$). Therefore, the remaining source of non-perturbative contributions would be the gluon condensates [see Fig(3)]. The calculation of such contribution

is lengthy but standard. For the nonperturbative part in Borel scheme, we get

$$\Pi^{non-pert} = M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle C_{G^2}, \quad (57)$$

where C_{G^2} is the Wilson coefficient of the gluon condensates that is defined as

$$C_{G^2} = C_{G^2}^a + C_{G^2}^b + C_{G^2}^c + C_{G^2}^d + C_{G^2}^e + C_{G^2}^f. \quad (58)$$

The explicit expression of the $C_{G^2}^i$ is given in the Appendix.

From the same procedure as presented in the previous section, we get the following expression for the form factor under consideration:

$$T_2(0) = \frac{(m_b + m_c) e^{m_{B_c}^2/M_1^2} e^{m_{D_{s1}}^2/M_2^2}}{i f_{D_{s1}} f_{B_c} m_{D_{s1}} m_{B_c}^2 (m_{B_c}^2 - m_{D_{s1}}^2)} \left[\frac{-1}{(2\pi)^2} \int \int ds' dt e^{-s'/M_1^2} e^{-t/M_2^2} \rho^{\text{pert}}(s', t) \right. \\ \left. + M_1^2 M_2^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle C_{G^2} \right], \quad (59)$$

where s'_0 and t_0 are the continuum thresholds in the initial and final channels.

V. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the form factors as well as estimating the branching ratio in WA and EP channels as well as the total branching fraction. For this aim, we use the quark and mesons' masses as $m_c = (1.275 \pm 0.015) \text{ GeV}$, $m_s \simeq 142 \text{ MeV}$ [14], $m_b = (4.7 \pm 0.1) \text{ GeV}$ [15], $m_{D_{s1}} = (2459.6 \pm 0.6) \text{ MeV}$, $m_{B_c} = (6.277 \pm 0.006) \text{ GeV}$ [16]. For the values of the decay constants, we use $f_{D_{s1}} = (225 \pm 25) \text{ MeV}$ and $f_{B_c} = (350 \pm 25) \text{ MeV}$ [17–19]. The values of the condensates are [15]: $\langle \bar{\psi}\psi|_{\mu=1\text{GeV}} \rangle = -(240 \pm 10 \text{ MeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{\psi}\psi \rangle$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012$. The parameters entered the photon DAs are also taken as $\chi = 3.15 \pm 0.3 \text{ GeV}^{-2}$, $k = 0.2$, $k^+ = 0$, $\zeta_1 = 0.4$, $\zeta_1^+ = 0$, $\zeta_2 = 0.3$, $\zeta_2^+ = 0$, $f_{3\gamma} = -(4 \pm 2) \times 10^{-3} \text{ GeV}^2$, $\omega_\gamma^A = -2.1 \pm 1.0$ and $\omega_\gamma^V = 3.8 \pm 1.8$ [12, 13, 20]. The remaining parameters are chosen as $|V_{cs}| = 0.957 \pm 0.017$, $|V_{cb}| = 0.0416 \pm 0.0006$, $|V_{tb}| = 0.77_{-0.24}^{+0.18}$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$, $C_7(\mu = m_c) = -0.0068 - 0.02i$, and $\tau_{B_c} = 0.52 \times 10^{-12} \text{ s}$.

The sum rules for the form factors contain also continuum thresholds and Borel mass parameters as auxiliary objects. We should find their working region such that the physical observables be practically independent of them. The continuum thresholds are not totally arbitrary but they are correlated with the energy of the first excited state in the initial and

final mesonic channels. Our numerical results show that, the results depend weakly on the thresholds in the intervals, $s_0 = t_0 = (6 - 8)GeV^2$ and $s'_0 = (45 - 50)GeV^2$. The working regions for the Borel parameters are obtained demanding that not only the contribution of the higher states and continuum are effectively suppressed but also the contribution of the higher order operators and higher twist DAs are small, i.e., series of the sum rules are convergent. These conditions lead to the intervals $6 GeV^2 \leq M_B^2 \leq 12 GeV^2$, $10 GeV^2 \leq M_1^2 \leq 30 GeV^2$ and $5 GeV^2 \leq M_2^2 \leq 12 GeV^2$ for the Borel mass parameters.

Now, we proceed to find the fit functions of the form factors using the aforesaid working regions for the auxiliary parameters as well as other input parameters. Here, we would like to mention that for the decay rates we need the values of the form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ at $Q^2 = m_{B_c}^2$, for $F_V^{(B_c)}$ and $F_A^{(B_c)}$ at $p^2 = m_{D_{s1}}^2$ and for T_2 at $q^2 = 0$. However we determine their fit functions in general then give the their values at fixed points. The fit functions for the form factors $F_V^{(D_{s1})}$ and $F_A^{(D_{s1})}$ are given as

$$f(Q^2) = \frac{f(0)}{1 + a\frac{Q^2}{m_{D_{s1}}^2} + b(\frac{Q^2}{m_{D_{s1}}^2})^2}, \quad (60)$$

where $f(0)$, a and b are the fit parameters whose values are given as:

form factors	$f(0)$	a	b
$F_V^{(D_{s1})}(Q^2)$	0.134	0.145	-0.008
$F_A^{(D_{s1})}(Q^2)$	-1.355	3.217	-0.205

The values of these form factors at $Q^2 = m_{B_c}^2$ are obtained as:

$$\begin{aligned} F_V^{(D_{s1})}(Q^2 = m_{B_c}^2) &= 0.083 \\ F_A^{(D_{s1})}(Q^2 = m_{B_c}^2) &= -0.102 \end{aligned} \quad (61)$$

Also the fit functions for the form factors $F_{A,V}^{(B_c)}$ are as follows [4]:

$$\begin{aligned} F_V^{(B_c)}(p^2) &= \frac{F_V(0)}{1 - p^2/m_1^2} \\ F_A^{(B_c)}(p^2) &= \frac{F_A(0)}{1 - p^2/m_2^2} \end{aligned} \quad (62)$$

where the fit parameters are given by

$F_V(0) = 0.44\text{GeV}$	$m_1^2 = 43.1\text{GeV}^2$
$F_A(0) = 0.21\text{GeV}$	$m_2^2 = 48.0\text{GeV}^2$

The values of the form factors $F_{V,A}^{(B_c)}$ obtained at $p^2 = m_{D_{s1}}^2$ are given as:

$$\begin{aligned} F_V^{(B_c)}(p^2 = m_{D_{s1}}^2) &= 0.512\text{GeV} \\ F_A^{(B_c)}(p^2 = m_{D_{s1}}^2) &= 0.240\text{GeV} \end{aligned} \quad (63)$$

For the form factor induced by the electromagnetic penguin at $q^2 = 0$, we obtain

$$T_2(0) = -0.298 \quad (64)$$

At the end of this section, we would like to calculate the total decay width and branching ratios. Using the amplitudes of each decay mode, we find the following expressions for the total decay rates at fixed points in different channels:

$$\begin{aligned} \Gamma^{(WA)}(B_c \rightarrow D_{s1}\gamma) &= \frac{G_F^2 \alpha |V_{cb} V_{cs}^*|^2}{16} \left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}} \right)^3 \\ &\times \left\{ f_{B_c}^2 \left[(F_A^{(D_{s1})})^2 + (F_V^{(D_{s1})})^2 \right] + 2f_{B_c} f_{D_{s1}} F_V^{(B_c)} F_V^{(D_{s1})} \frac{m_{D_{s1}}}{m_{B_c}^2} \right. \\ &\left. + f_{D_{s1}}^2 m_{D_{s1}}^2 \left[\frac{(F_A^{(B_c)})^2}{m_{B_c}^4} + \frac{(F_V^{(B_c)})^2}{m_{B_c}^4} \right] \right\}, \end{aligned} \quad (65)$$

$$\begin{aligned} \Gamma^{(EP)}(B_c \rightarrow D_{s1}\gamma) &= \frac{G_F^2 \alpha |C_7|^2 |V_{tb} V_{ts}^*|^2}{1024\pi^4} \left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}} \right)^3 \\ &\times \left(17m_b^2 + 30m_b m_s + 17m_s^2 \right) [T_2(0)]^2, \end{aligned} \quad (66)$$

$$\begin{aligned} \Gamma^{(total)}(B_c \rightarrow D_{s1}\gamma) &= \frac{G_F^2 \alpha}{1024\pi^4} \left(\frac{m_{B_c}^2 - m_{D_{s1}}^2}{m_{B_c}} \right)^3 \left\{ 64\pi^4 |V_{cb} V_{cs}^*|^2 \left[f_{B_c}^2 \{ (F_A^{(D_{s1})})^2 + (F_V^{(D_{s1})})^2 \} \right. \right. \\ &+ 2f_{B_c} f_{D_{s1}} F_V^{(B_c)} F_V^{(D_{s1})} \frac{m_{D_{s1}}}{m_{B_c}^2} + f_{D_{s1}}^2 m_{D_{s1}}^2 \left\{ \frac{(F_A^{(B_c)})^2}{m_{B_c}^4} + \frac{(F_V^{(B_c)})^2}{m_{B_c}^4} \right\} \left. \right] \\ &+ |C_7|^2 |V_{tb} V_{ts}^*|^2 (17m_b^2 - 30m_b m_s + 17m_s^2) [T_2(0)]^2 \\ &+ 16\pi^2 T_2(0) |V_{cb} V_{cs}^*| |V_{tb} V_{ts}^*| \left[f_{D_{s1}} m_{D_{s1}} X \left\{ 4 \frac{F_A^{(B_c)}}{m_{B_c}^2} (m_b - m_s) \right. \right. \\ &+ \frac{F_V^{(B_c)}}{m_{B_c}^2} (m_b + m_s) \left. \right\} + f_{B_c} \{ F_V^{(D_{s1})} (m_b + m_s) X \\ &\left. \left. - 4F_A^{(D_{s1})} (m_b - m_s) Y \right\} \right] \left. \right\}, \end{aligned} \quad (67)$$

where X and Y , are the real and imaginary parts of the Wilson coefficient C_7 , respectively. In these formulas, the fixed point values of the form factors discussed in the above are used.

Finally the numerical values of the corresponding branching ratios for the radiative decay under consideration are obtained as follows:

$$\begin{aligned}\mathbf{B}^{(EP)}(B_c \rightarrow D_{s1}\gamma) &= 1.765 \times 10^{-8}, \\ \mathbf{B}^{(WA)}(B_c \rightarrow D_{s1}\gamma) &= 3.034 \times 10^{-5}, \\ \mathbf{B}^{(total)}(B_c \rightarrow D_{s1}\gamma) &= 3.136 \times 10^{-5}.\end{aligned}\tag{68}$$

The order of the total branching ratio indicates that this decay channel can be detected at LHC. Any measurement in this respect and the comparison of the obtained data with our predictions in the present work can give valuable information about the nature and internal structure of the participating particles especially the D_{s1} meson.

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VI. APPENDIX

The explicit expressions for $C_{G^2}^i$ are given as follows:

$$\begin{aligned}C_{G^2}^a = m_b \{ & 2m_s^3(I_0[1, 3, 1] - 3m_b^2 I_0[1, 4, 1] + 2m_b^4 I_0[1, 5, 1]) \\ & - 2m_b m_c^2 [I_0[1, 3, 1] + (-3m_b^2 + m_s^2) I_0[1, 4, 1] + 2m_b^2 (m_b - m_s)(m_b + m_s) I_0[1, 5, 1]] \\ & + 2m_c (I_0[1, 2, 1] - 4m_b^2 I_0[1, 3, 1] - m_b m_s I_0[1, 3, 1] + m_s^2 I_0[1, 3, 1] + 5m_b^4 I_0[1, 4, 1] \\ & + 3m_b^3 m_s I_0[1, 4, 1] - 3m_b^2 m_s^2 I_0[1, 4, 1] - m_b m_s^3 I_0[1, 4, 1] \\ & + 2m_b^3 (-m_b + m_s)(m_b + m_s)^2 I_0[1, 5, 1]) + 2m_s [I_0[1, 2, 1] - 2m_b^6 I_0[1, 5, 1] + I_0^{[0,1]}[1, 3, 1] \\ & + m_b^4 (5I_0[1, 4, 1] + 2I_0^{[0,1]}[1, 5, 1]) + m_b^2 (-4I_0[1, 3, 1] - 3I_0^{[0,1]}[1, 4, 1] + 3I_0^{[1,0]}[1, 3, 1]) \\ & + m_b m_s^2 [I_0^{[0,1]}[1, 4, 1] + 3I_0^{[1,0]}[1, 4, 1] - 2m_b^2 (I_0^{[0,1]}[1, 5, 1] + 3I_0^{[1,0]}[1, 5, 1])] \\ & + m_b (3I_0^{[0,1]}[1, 3, 1] + I_0^{[0,2]}[1, 4, 1] + I_0^{[1,0]}[1, 3, 1] + 2m_b^4 (3I_0^{[0,1]}[1, 5, 1] + I_0^{[1,0]}[1, 5, 1]) \\ & - m_b^2 (9I_0^{[0,1]}[1, 4, 1] + 2I_0^{[0,2]}[1, 5, 1] + 3I_0^{[1,0]}[1, 4, 1] - 2I_0^{[2,0]}[1, 4, 1]) \\ & - 4I_3^{[0,1]}[1, 5, 1] + 4I_3^{[1,0]}[1, 3, 1]) \},\end{aligned}\tag{69}$$

$$\begin{aligned}
C_{G^2}^b = & -6m_b m_s^2 I_0[1, 1, 1] + 6m_c m_s^2 I_0[1, 1, 1] - 5m_b^3 m_s^2 I_0[1, 1, 2] - 8m_b m_c m_s^2 I_0[1, 1, 2] \\
& + 5m_b^2 m_c m_s^2 I_0[1, 1, 2] + m_c^2 m_s^2 I_0[1, 1, 2] + 2m_b m_s^3 I_0[1, 1, 2] - 2m_c m_s^3 I_0[1, 1, 2] \\
& - 12m_b m_c m_s^3 I_0[1, 1, 2] + 12m_c^2 m_s^3 I_0[1, 1, 2] + 11m_b m_s^4 I_0[1, 1, 2] - 11m_c m_s^4 I_0[1, 1, 2] \\
& - 7m_b^3 m_c m_s^2 I_0[1, 1, 3] + 7m_b^2 m_c^2 m_s^2 I_0[1, 1, 3] + m_b^3 m_s^3 I_0[1, 1, 3] - m_b^2 m_c m_s^3 I_0[1, 1, 3] \\
& - 10m_b^3 m_c m_s^3 I_0[1, 1, 3] + 10m_b^2 m_c^2 m_s^3 I_0[1, 1, 3] + 6m_b^3 m_s^4 I_0[1, 1, 3] \\
& + 17m_b m_c m_s^4 I_0[1, 1, 3] - 6m_b^2 m_c m_s^4 I_0[1, 1, 3] - 17m_c^2 m_s^4 I_0[1, 1, 3] + m_b m_s^5 I_0[1, 1, 3] \\
& - m_c m_s^5 I_0[1, 1, 3] + 22m_b m_c m_s^5 I_0[1, 1, 3] - 22m_c^2 m_s^5 I_0[1, 1, 3] + 6m_c m_s^6 I_0[1, 1, 3] \\
& - 2m_b m_s^7 I_0[1, 1, 3] + m_b^3 I_0[1, 1, 4] + m_c I_0[1, 1, 4] + m_b m_c I_0[1, 1, 4] - m_b^2 m_c I_0[1, 1, 4] \\
& - m_b^3 m_s I_0[1, 1, 4] - m_c m_s I_0[1, 1, 4] + 2m_b m_c m_s I_0[1, 1, 4] + m_b^2 m_c m_s I_0[1, 1, 4] \\
& + 10m_b^3 m_c m_s^4 I_0[1, 1, 4] - 10m_b^2 m_c^2 m_s^4 I_0[1, 1, 4] + 2m_b^3 m_s^5 I_0[1, 1, 4] - 2m_b^2 m_c m_s^5 I_0[1, 1, 4] \\
& + 12m_b^3 m_c m_s^5 I_0[1, 1, 4] - 12m_b^2 m_c^2 m_s^5 I_0[1, 1, 4] - 10m_b m_c m_s^6 I_0[1, 1, 4] - 12m_b m_c m_s^7 I_0[1, 1, 4] \\
& + 12m_c^2 m_s^7 I_0[1, 1, 4] - m_c I_0[1, 1, 5] - m_b m_c I_0[1, 1, 5] + m_b^3 m_c I_0[1, 1, 5] - m_b^2 m_c^2 I_0[1, 1, 5] \\
& + m_c m_s I_0[1, 1, 5] - 2m_b m_c m_s I_0[1, 1, 5] + 2m_b^3 m_c m_s I_0[1, 1, 5] - 2m_b^2 m_c^2 m_s I_0[1, 1, 5] \\
& - 6m_s^7 I_0^{[0,1]}[1, 1, 2] - m_b I_0^{[0,1]}[1, 1, 4] + m_s I_0^{[0,1]}[1, 1, 4] + 21/2 m_s^2 I_0^{[0,1]}[1, 1, 4] \\
& + 5m_b m_s^2 I_0^{[0,1]}[1, 1, 4] - m_b m_s^3 I_0^{[0,1]}[1, 1, 4] - 13/2 m_s^4 I_0^{[0,1]}[1, 1, 4] - 3/2 I_0^{[0,1]}[1, 1, 5] \\
& + 3/2 m_b^2 I_0^{[0,1]}[1, 1, 5] + m_b m_s I_0^{[0,1]}[1, 1, 5] - 21/2 m_b^2 m_s^2 I_0^{[0,1]}[1, 1, 5] + 15m_s^3 I_0^{[0,1]}[1, 1, 5] \\
& - 15m_b^2 m_s^3 I_0^{[0,1]}[1, 1, 5] - 6m_b m_s^4 I_0^{[0,1]}[1, 1, 5] + 15m_b^2 m_s^4 I_0^{[0,1]}[1, 1, 5] - 7m_s^5 I_0^{[0,1]}[1, 1, 5] \\
& - 2m_b m_s^5 I_0^{[0,1]}[1, 1, 5] + 18m_b^2 m_s^5 I_0^{[0,1]}[1, 1, 5] - m_s I_0^{[0,2]}[1, 1, 2] - 6m_s^5 I_0^{[0,2]}[1, 1, 2] \\
& + 7/2 m_s^2 I_0^{[0,2]}[1, 1, 3] + 5m_s^3 I_0^{[0,2]}[1, 1, 3] + 7/2 m_s^2 I_0^{[1,0]}[1, 1, 2] + 3/2 I_0^{[1,0]}[1, 1, 3] \\
& + m_b I_0^{[1,0]}[1, 1, 3] + 3m_s I_0^{[1,0]}[1, 1, 3] - 5m_b m_s^2 I_0^{[1,0]}[1, 1, 3] - 7/2 m_b^2 m_s^2 I_0^{[1,0]}[1, 1, 3] \\
& + m_b m_s^3 I_0^{[1,0]}[1, 1, 3] - 5m_s^4 I_0^{[1,0]}[1, 1, 3] - 6m_s^5 I_0^{[1,0]}[1, 1, 3] + 6m_b^2 m_s^5 I_0^{[1,0]}[1, 1, 3] \\
& - 18m_s^7 I_0^{[1,0]}[1, 1, 3] + 1/2 m_b^2 I_0^{[1,0]}[1, 1, 4] - m_b m_s I_0^{[1,0]}[1, 1, 4] + 5m_s^3 I_0^{[1,0]}[1, 1, 4] \\
& + 51/2 m_s^4 I_0^{[1,0]}[1, 1, 4] + 6m_b m_s^4 I_0^{[1,0]}[1, 1, 4] + 33m_s^5 I_0^{[1,0]}[1, 1, 4] - 5m_s^3 I_0^{[2,0]}[1, 1, 3] \\
& + 1/2 I_0^{[2,0]}[1, 1, 4] + m_s I_0^{[2,0]}[1, 1, 4] + 6m_s^5 I_0^{[2,0]}[1, 1, 4], \tag{70}
\end{aligned}$$

$$\begin{aligned}
C_{G^2}^c = & 1/6\{2m_c^6 m_s^2 I_0[3, 1, 2] + 2m_c^5 m_s (I_0[3, 1, 1] + m_s^2 I_0[3, 1, 2] - I_0[3, 2, 1]) \\
& + 2m_b^5 [-m_c m_s^2 I_0[3, 1, 2] + m_s^3 I_0[3, 1, 2] + m_s (-I_0[3, 1, 1] + m_c^2 I_0[3, 1, 2] + I_0^{[0,1]}[3, 1, 1]) \\
& + m_c (m_c^2 I_0[3, 2, 2] + I_0^{[0,1]}[3, 1, 2]) - 3I_0^{[0,1]}[3, 2, 1] + 3m_s^2 I_0^{[0,1]}[3, 2, 1] + I_0^{[0,1]}[3, 2, 2] \\
& - 2m_s^2 I_0^{[0,1]}[3, 2, 2] + m_s^4 I_0 * [0, 1][3, 2, 2] + 3I_0^{[0,2]}[3, 1, 2] - 2I_0^{[0,2]}[3, 2, 1] + 2m_s^2 I_0^{[0,2]}[3, 2, 2] \\
& + I_0^{[0,3]}[3, 2, 2] + 3I_0^{[1,0]}[3, 1, 1] - 6m_s^2 I_0^{[1,0]}[3, 1, 1] - I_0^{[1,0]}[3, 1, 2] \\
& + 2m_c^3 m_s \{I_0[3, 2, 1] + m_s^4 I_0[3, 2, 1] - I_0[3, 2, 2] - 4I_0^{[0,1]}[3, 1, 2] + 3I_0^{[0,1]}[3, 2, 1] + 3I_0^{[1,0]}[3, 1, 1] \\
& - 2I_0^{[1,0]}[3, 2, 1] + m_s^2 [-2I_0[3, 1, 2] + I_0[3, 2, 2] - 3(I_0^{[0,1]}[3, 2, 2] + I_0^{[1,0]}[3, 2, 1])]\} \\
& + 3m_s^4 I_0^{[1,0]}[3, 2, 2] - I_0^{[1,1]}[3, 1, 1] + I_0^{[1,1]}[3, 1, 2] - m_s^4 I_0^{[1,1]}[3, 1, 2] \\
& - 2m_b^3 [m_s^5 I_0[3, 2, 1] - m_c m_s^4 I_0[3, 2, 2] - 2m_s^3 (I_0[3, 1, 1] - I_0^{[0,1]}[3, 2, 1] + I_0^{[1,0]}[3, 2, 1]) \\
& + m_c m_s^2 (2m_c^2 I_0[3, 2, 2] - I_0^{[0,1]}[3, 2, 2] + 3I_0^{[1,0]}[3, 2, 2]) - m_c (m_c^4 I_0[3, 1, 2] - 2I_0^{[0,1]}[3, 2, 2] \\
& + I_0^{[1,0]}[3, 1, 2] + m_c^2 (I_0[3, 1, 2] - 2I_0[3, 2, 2] - 3(I_0^{[0,1]}[3, 2, 2] + I_0^{[1,0]}[3, 2, 2])) + I_0^{[1,1]}[3, 2, 1]) \\
& + m_s ((1 + 2m_c^2) I_0[3, 2, 2] + 2I_0^{[0,1]}[3, 1, 1] - 2I_0^{[0,1]}[3, 2, 1] + 4m_c^2 I_0^{[0,1]}[3, 2, 2] + I_0^{[0,2]}[3, 1, 2] \\
& + 2m_c^2 I_0^{[1,0]}[3, 1, 1] + 2I_0^{[1,0]}[3, 2, 2] - 2I_0^{[1,1]}[3, 2, 2]) + 2m_s^2 I_0^{[1,1]}[3, 2, 2] + m_b^4 (2m_c^4 I_0[3, 1, 2] \\
& + 2m_c^3 m_s I_0[3, 2, 2] + 2m_c m_s (m_s^2 I_0[3, 1, 2] - I_0[3, 2, 2] + I_0^{[0,1]}[3, 1, 1]) - 3I_0^{[0,1]}[3, 2, 2] \\
& + 3I_0^{[0,2]}[3, 2, 2] - I_0^{[1,0]}[3, 1, 2] + m_c^2 (-2m_s^2 I_0[3, 1, 1] - 3I_0^{[0,1]}[3, 2, 2] + I_0^{[1,0]}[3, 1, 2]) \\
& + m_s^2 (3I_0^{[0,1]}[3, 1, 2] + I_0^{[1,0]}[3, 2, 2]) + I_0^{[1,1]}[3, 2, 2]) + 2m_c m_s [-I_0^{[0,1]}[3, 2, 1] \\
& + I_0^{[0,2]}[3, 2, 2] + I_0^{[1,0]}[3, 2, 2] - I_0^{[1,1]}[3, 1, 2] + m_s^2 (I_0^{[0,1]}[3, 2, 2] - 2I_0^{[1,0]}[3, 1, 1] + I_0^{[1,1]}[3, 2, 2])] \\
& - I_0^{[1,2]}[3, 2, 2] - I_0^{[1,3]}[3, 1, 2] + 2I_0^{[2,0]}[3, 1, 2] + 6m_s^2 I_0^{[2,0]}[3, 1, 2] - m_c^4 (2m_c^4 I_0[3, 1, 1] \\
& + 2I_0^{[0,1]}[3, 2, 2] + I_0^{[0,2]}[3, 2, 2] - 9I_0^{[1,0]}[3, 1, 2] + 7I_0^{[1,0]}[3, 2, 2] + m_s^2 (2I_0[3, 1, 2] \\
& - 2I_0[3, 2, 1] + 7I_0^{[0,1]}[3, 2, 2] + 9I_0^{[1,0]}[3, 2, 2]) - I_0^{[2,0]}[3, 2, 2]) - 3I_0^{[2,0]}[3, 2, 2] - 2m_s^2 I_0^{[2,0]}[3, 2, 2] \\
& - 3m_s^4 I_0^{[2,0]}[3, 2, 2] - 2m_b (m_c^5 m_s^2 I_0[3, 2, 2] + m_c^4 m_s (m_s^2 I_0[3, 2, 2] + I_0^{[0,1]}[3, 1, 2]) \\
& - m_c^3 ((1 + 2m_s^2) I_0[3, 1, 2] - I_0[3, 2, 2] + 3I_0^{[0,1]}[3, 1, 2] - 2I_0^{[0,1]}[3, 2, 2] - 4I_0^{[1,0]}[3, 1, 2] \\
& + m_s^2 (-I_0[3, 2, 1] + I_0^{[0,1]}[3, 2, 1] + 5I_0^{[1,0]}[3, 2, 1]) + 3I_0^{[1,0]}[3, 2, 2]) + m_c^2 m_s ((1 + m_s^4) I_0[3, 1, 2] \\
& - (1 + 2m_s^2) I_0[3, 2, 1] - 4I_0^{[0,1]}[3, 1, 2] + 2I_0^{[0,1]}[3, 2, 1] - 2m_s^2 I_0^{[0,1]}[3, 2, 1] - 3I_0^{[0,2]}[3, 2, 2] \\
& - 4m_s^2 I_0^{[1,0]}[3, 1, 2] + 3I_0^{[2,0]}[3, 2, 1]) + m_c (-I_0^{[0,1]}[3, 2, 2] + m_s^4 (I_0[3, 2, 1] - I_0^{[1,0]}[3, 2, 1]) \\
& + I_0^{[1,0]}[3, 2, 2] + I_0^{[1,1]}[3, 1, 2] - I_0^{[2,0]}[3, 2, 2] + m_s^2 (I_0[3, 1, 1] - I_0[3, 2, 1] + I_0^{[0,1]}[3, 2, 1] \\
& - 3I_0^{[1,0]}[3, 1, 2] + I_0^{[1,0]}[3, 2, 1] - I_0^{[1,1]}[3, 1, 2] + 2I_0^{[2,0]}[3, 2, 2])) - m_s (I_0^{[0,2]}[3, 2, 2] \\
& + 2I_0^{[1,0]}[3, 1, 2] + m_s^4 (I_0[3, 1, 2] - I_0^{[1,0]}[3, 2, 2]) - I_0^{[1,0]}[3, 2, 2] - I_0^{[2,0]}[3, 1, 2]
\end{aligned} \tag{71}$$

$$\begin{aligned}
& +m_s^2(I_0[3, 2, 1] - 2I_0[3, 2, 2] + 2I_0^{[0,1]}[3, 1, 2] + 2I_0^{[1,0]}[3, 1, 1] - 2I_0^{[1,1]}[3, 2, 2] + I_0^{[2,0]}[3, 2, 1]) \\
& + I_0^{[2,1]}[3, 1, 2])) - 3m_s^2I_0^{[2,1]}[3, 2, 1] - m_b^2(2m_c^6I_0[3, 2, 2] + I_0^{[0,1]}[3, 1, 2] - 6I_0^{[0,1]}[3, 2, 1] \\
& - 2I_0^{[0,2]}[3, 1, 2] + 6I_0^{[0,2]}[3, 2, 2] + I_0^{[0,3]}[3, 2, 2] + 3m_s^4I_0^{[1,0]}[3, 1, 2] + 2m_c^3m_s(I_0[3, 1, 2] \\
& + 2(-1 + m_s^2)I_0[3, 2, 1] - 5I_0^{[0,1]}[3, 2, 2] - I_0^{[1,0]}[3, 2, 1]) + 3I_0^{[1,0]}[3, 2, 1] - 2I_0^{[1,0]}[3, 2, 2] \\
& - 7m_c^4I_0^{[1,0]}[3, 2, 2] + 2m_cm_s(I_0[3, 1, 2] + (-1 + m_s^2)I_0[3, 2, 1] - 2m_s^2I_0[3, 2, 2] \\
& + m_s^4I_0[3, 2, 2] + 3m_s^2I_0^{[0,1]}[3, 1, 2] + 2I_0^{[0,2]}[3, 1, 2] - m_s^2I_0^{[1,0]}[3, 2, 1] - I_0^{[1,1]}[3, 2, 1]) + 2I_0^{[1,1]}[3, 2, 1] \\
& - 3I_0^{[1,2]}[3, 2, 2] + 2I_0^{[2,0]}[3, 2, 1] - 2m_s^2(I_0^{[0,1]}[3, 1, 1] - I_0^{[0,2]}[3, 2, 2] - I_0^{[1,0]}[3, 1, 2] + 3I_0^{[1,0]}[3, 2, 1] \\
& + I_0^{[2,0]}[3, 2, 1]) - 2m_c^2(m_s^2I_0[3, 2, 2] + m_s^4I_0[3, 2, 2] + 2I_0^{[0,1]}[3, 2, 2] - 5I_0^{[0,2]}[3, 2, 2] - 2I_0^{[1,0]}[3, 2, 2] \\
& - 7I_0^{[1,1]}[3, 2, 1] - I_0^{[2,0]}[3, 2, 2]) - 2I_0^{[2,1]}[3, 2, 2]) + I_0^{[2,1]}[3, 2, 2] + m_c^2(-2I_0^{[0,1]}[3, 2, 2] \\
& + 10I_0^{[0,2]}[3, 1, 2] - 2I_0^{[0,2]}[3, 2, 1] + I_0^{[1,0]}[3, 2, 1] - m_s^4(I_0^{[0,1]}[3, 1, 1] - 3I_0^{[1,0]}[3, 2, 2]) + I_0^{[1,0]}[3, 2, 2] \\
& - 14I_0^{[1,1]}[3, 1, 2] + 14I_0^{[1,1]}[3, 2, 2] + 3I_0^{[1,2]}[3, 2, 2] + 2I_0^{[2,0]}[3, 1, 2] - 10I_0^{[2,0]}[3, 2, 1] \\
& + 2m_s^2(I_0^{[0,1]}[3, 1, 1] + I_0^{[0,1]}[3, 1, 2] + I_0^{[0,2]}[3, 1, 2] - 2I_0^{[1,0]}[3, 1, 2] + 7I_0^{[1,1]}[3, 2, 2] + 5I_0^{[2,0]}[3, 2, 2]) \\
& - 3I_0^{[2,1]}[3, 2, 2] - 3I_0^{[3,0]}[3, 2, 2]) - I_0^{[3,0]}[3, 2, 2] + m_s^2I_0^{[3,0]}[3, 2, 2] + I_0^{[3,1]}[3, 2, 2]\},
\end{aligned}$$

$$\begin{aligned}
C_{G^2}^d = & 1/12\{2m_c^4((-1 + m_s^2)I_0[3, 1, 2] + I_0[3, 2, 1]) + 2m_c^3m_s(I_0[3, 1, 1] + m_s^2I_0[3, 2, 1]) \\
& + 24m_b^7(m_c + m_s)I_0[3, 2, 2] + I_0^{[0,1]}[3, 2, 2] + 2m_b^6(8m_cm_sI_0[3, 1, 1] + 8m_c^2I_0[3, 2, 2] \\
& + 3I_0^{[0,1]}[3, 1, 2] + I_0^{[1,0]}[3, 1, 1]) + 6m_b^5(2m_c^3I_0[3, 1, 1] + 2m_c^2m_sI_0[3, 2, 2] - 2m_s(-2I_0[3, 1, 2] \\
& + (5 + 2m_s^2)I_0[3, 2, 1] + 2I_0^{[0,1]}[3, 2, 2]) + m_c(-2(3 + 2m_s^2)I_0[3, 2, 2] - 9I_0^{[0,1]}[3, 2, 1] \\
& + I_0^{[1,0]}[3, 1, 2])) + 3I_0^{[1,0]}[3, 2, 1] - 3m_s^2I_0^{[1,0]}[3, 2, 1] - I_0^{[1,0]}[3, 2, 2] - 2m_cm_s(2I_0^{[0,1]}[3, 1, 2] \\
& - I_0^{[1,0]}[3, 1, 1] - I_0^{[1,0]}[3, 1, 2] + m_s^2(I_0[3, 2, 2] + I_0^{[1,0]}[3, 2, 2])) + m_s^2I_0^{[1,1]}[3, 1, 2] \\
& + 2I_0^{[1,1]}[3, 2, 2] + I_0^{[1,2]}[3, 1, 2] + 3m_s^2I_0^{[2,0]}[3, 1, 2] - m_c^2(2(-1 + m_s^2)I_0[3, 1, 2] + 2I_0[3, 2, 2] \\
& + m_s^2I_0^{[0,1]}[3, 1, 1] + 2I_0^{[0,1]}[3, 2, 2] + I_0^{[0,2]}[3, 2, 2] - 5I_0^{[1,0]}[3, 2, 1] + 3I_0^{[1,0]}[3, 2, 2] \\
& + 5m_s^2I_0^{[1,0]}[3, 2, 2] - I_0^{[2,0]}[3, 2, 2]) - I_0^{[2,0]}[3, 2, 2] + m_b^4(4m_c^4I_0[3, 2, 1] + 4m_c^3m_sI_0[3, 2, 2] \\
& + 2I_0^{[0,1]}[3, 2, 1] - 15I_0^{[0,1]}[3, 2, 2] - 2I_0^{[0,2]}[3, 1, 1] - 4m_cm_s(4(-1 + m_s^2)I_0[3, 2, 1] \\
& + 5I_0^{[0,1]}[3, 2, 2] - 4I_0^{[1,0]}[3, 1, 2]) + 6I_0^{[1,0]}[3, 2, 2] - 6m_s^2I_0^{[1,0]}[3, 2, 2] \\
& - 2m_c^2((6 + 8m_s^2)I_0[3, 2, 2] + 3(I_0^{[0,1]}[3, 2, 2] + I_0^{[1,0]}[3, 2, 2])) + 6I_0^{[1,1]}[3, 1, 2] + 4I_0^{[2,0]}[3, 2, 2]) \\
& + 3m_b(2m_c^2m_s(-I_0[3, 2, 1] + m_s^2I_0[3, 2, 2] - 2I_0^{[0,1]}[3, 2, 2] + 2I_0^{[1,0]}[3, 2, 2]) \\
& + m_c(-4(-1 + m_s^2)I_0[3, 1, 2] - 4I_0[3, 2, 2] - 3m_s^2I_0^{[0,1]}[3, 1, 1] + 3I_0^{[0,1]}[3, 2, 1])
\end{aligned} \tag{72}$$

$$\begin{aligned}
& -9I_0^{[0,1]}[3, 2, 2] - 3I_0^{[0,2]}[3, 1, 2] + 5I_0^{[1,0]}[3, 1, 2] + I_0^{[1,0]}[3, 2, 1] - 5m_s^2 I_0^{[1,0]}[3, 2, 2] \\
& + 3I_0^{[2,0]}[3, 1, 2]) - 4m_s(I_0[3, 1, 2] + (-1 + m_s^2)I_0[3, 2, 2] + I_0^{[0,1]}[3, 2, 2] \\
& + I_0^{[1,0]}[3, 1, 2] - 2I_0^{[1,0]}[3, 2, 1] - m_s^2 I_0^{[1,0]}[3, 2, 2] - I_0^{[1,1]}[3, 2, 1] + I_0^{[2,0]}[3, 2, 2])) \\
& - 3m_b^3(4m_c^3(m_s^2 I_0[3, 1, 1] - I_0[3, 2, 2]) + 2m_c^2 m_s((3 + 2m_s^2)I_0[3, 1, 2] - 4I_0^{[0,1]}[3, 2, 2] \\
& + 4I_0^{[1,0]}[3, 1, 2]) - 4m_s(-3I_0[3, 2, 1] + 4I_0[3, 2, 2] + 3I_0^{[0,1]}[3, 2, 2] + m_s^2(3I_0[3, 1, 2] \\
& - 2I_0^{[1,0]}[3, 2, 2]) - 4I_0^{[1,0]}[3, 2, 2] - 2I_0^{[1,1]}[3, 2, 2] + 2I_0^{[2,0]}[3, 2, 2]) + m_c(-16I_0[3, 1, 2] \\
& + 12I_0[3, 2, 2] - 27I_0^{[0,1]}[3, 1, 2] + 6I_0^{[0,1]}[3, 2, 1] - 6I_0^{[0,2]}[3, 2, 1] + 3I_0^{[1,0]}[3, 1, 1] \\
& - 2m_s^2(6I_0[3, 2, 1] + 3I_0^{[0,1]}[3, 2, 1] + 5I_0^{[1,0]}[3, 1, 2]) + 10I_0^{[1,0]}[3, 2, 2] + 6I_0^{[2,0]}[3, 2, 2])) \\
& + m_b^2(m_c^4(-6I_0[3, 1, 2] - 4m_s^2 I_0[3, 2, 1] + 4I_0[3, 2, 2]) + m_c^3(4m_s I_0[3, 1, 2] - 6m_s I_0[3, 2, 1] \\
& - 4m_s^3 I_0[3, 2, 2]) - 3I_0^{[0,1]}[3, 1, 1] + 12I_0^{[0,1]}[3, 1, 2] - 9I_0^{[1,0]}[3, 1, 2] + 4I_0^{[1,0]}[3, 2, 1] \\
& + 2m_c m_s(7I_0[3, 1, 1] + 9I_0^{[0,1]}[3, 2, 2] + 2m_s^2(3I_0[3, 1, 2] + I_0^{[1,0]}[3, 1, 2]) - 2I_0^{[1,0]}[3, 2, 1] \\
& - 6I_0^{[1,0]}[3, 2, 2]) + 9m_s^2 I_0^{[1,0]}[3, 2, 2] - 9I_0^{[1,1]}[3, 1, 1] - 2m_s^2 I_0^{[1,1]}[3, 2, 2] - 2I_0^{[1,2]}[3, 2, 1] \\
& + m_c^2(14I_0[3, 1, 2] - 12I_0[3, 2, 2] + 9I_0^{[0,1]}[3, 1, 2] - 2I_0^{[0,1]}[3, 2, 2] + 2I_0^{[0,2]}[3, 1, 2] \\
& - 10I_0^{[1,0]}[3, 1, 2] + 2m_s^2(6I_0[3, 2, 1] + I_0^{[0,1]}[3, 2, 1] + 5I_0^{[1,0]}[3, 1, 2]) + 9I_0^{[1,0]}[3, 2, 1] \\
& - 2I_0^{[2,0]}[3, 1, 2]) - 6I_0^{[2,0]}[3, 1, 2] + 6I_0^{[2,0]}[3, 2, 2] - 6m_s^2 I_0^{[2,0]}[3, 2, 2] + 2I_0^{[3,0]}[3, 1, 0]) \\
& - I_0^{[3,0]}[3, 2, 2]\},
\end{aligned}$$

$$\begin{aligned}
C_{G^2}^e = & 1/6\{-2m_c m_s(m_s^2(2I_0[1, 2, 2] - 3I_0[1, 3, 1]) - I_0[1, 3, 2] + 2m_s^4 I_0[1, 3, 2] + I_0[1, 3, 3]) \\
& + 4m_b^5(2m_c m_s^2 I_0[1, 2, 1] + 4m_s^3 I_0[1, 2, 1] - m_c I_0[1, 2, 3] - 2m_s I_0[1, 3, 3]) \\
& - 2m_c^2((1 + 2m_s^2)I_0[1, 2, 2] - 3m_s^2 I_0[1, 3, 1] - I_0[1, 3, 3] + 2m_s^4 I_0[1, 3, 3]) - 3I_0^{[0,1]}[1, 1, 3] \\
& + I_0^{[0,1]}[1, 2, 2] + 6m_s^2 I_0^{[0,1]}[1, 2, 3] + 2m_s^4 I_0^{[0,1]}[1, 2, 3] - 3m_s^2 I_0^{[0,1]}[1, 3, 2] + 2m_s^2 I_0^{[0,2]}[1, 1, 2] \\
& - I_0^{[0,2]}[1, 2, 3] - 9m_s^2 I_0^{[1,0]}[1, 1, 3] - I_0^{[1,0]}[1, 2, 2] + 6m_s^4 I_0^{[1,0]}[1, 2, 2] + 3I_0^{[1,0]}[1, 3, 1] \\
& - 2m_b^4(m_c(-2m_s I_0[1, 2, 1] + 4m_s^3 I_0[1, 2, 2]) + m_c^2(4m_s^2 I_0[1, 2, 1] - 2I_0[1, 3, 3]) \\
& + (3 - 6m_s^2)I_0^{[0,1]}[1, 3, 3] + I_0^{[1,0]}[1, 2, 2] - 2m_s^2 I_0^{[1,0]}[1, 3, 2]) - 2m_b^3[m_c(-3I_0[1, 2, 3] \\
& - 6m_s^2(I_0[1, 1, 2] - I_0[1, 3, 2]) + 4m_s^4 I_0[1, 3, 2] + 2I_0[1, 3, 3]) + 2m_s(4m_s^4 I_0[1, 2, 3] \\
& - I_0[1, 3, 3] - 2I_0^{[0,1]}[1, 2, 2] + m_s^2(-6I_0[1, 1, 2] + 6I_0[1, 3, 2] + 4I_0^{[0,1]}[1, 3, 3] - 4I_0^{[1,0]}[1, 1, 3]) \\
& + 2I_0^{[1,0]}[1, 3, 2]) + 2m_b\{m_c[(-1 + 2m_s^2)I_0[1, 2, 2] + 2m_s^4 I_0[1, 2, 3] - 3m_s^2 I_0[1, 3, 1] \\
& + I_0[1, 3, 2]] + 2m_s[2m_s^4 I_0[1, 2, 2] - I_0[1, 2, 3] + I_0[1, 3, 2] - I_0^{[0,1]}[1, 1, 3] + I_0^{[1,0]}[1, 2, 3] \} \quad (73)
\end{aligned}$$

$$\begin{aligned}
& +m_s^2(2I_0[1, 2, 2] - 3I_0[1, 3, 1] + 2I_0^{[0,1]}[1, 2, 3] - 2I_0^{[1,0]}[1, 3, 3]))\} + 2m_s^2I_0^{[1,0]}[1, 3, 3] \\
& - 2m_s^2I_0^{[2,0]}[1, 1, 3] + I_0^{[2,0]}[1, 3, 2] + m_b^2(2m_c^2(4m_s^4I_0[1, 1, 3] + 2I_0[1, 2, 3] - 6m_s^2(I_0[1, 1, 2] \\
& - I_0[1, 3, 2]) - 3I_0[1, 3, 2]) + 2m_cm_s(2I_0[1, 1, 2] - 6m_s^2(I_0[1, 1, 3] - I_0[1, 3, 2]) - 3I_0[1, 3, 3] \\
& + 4m_s^4I_0[1, 3, 3]) + (9 + 6m_s^2)I_0^{[0,1]}[1, 2, 3] - 2I_0^{[0,1]}[1, 3, 2] - 18m_s^2I_0^{[0,1]}[1, 3, 3] - 4m_s^4I_0^{[0,1]}[1, 3, 3] \\
& - 4m_s^2I_0^{[0,2]}[1, 2, 1] + 2I_0^{[0,2]}[1, 3, 3] - 6I_0^{[1,0]}[1, 1, 3] - 6m_s^2I_0^{[1,0]}[1, 2, 2] + 18m_s^2I_0^{[1,0]}[1, 2, 3] \\
& - 12m_s^4I_0^{[1,0]}[1, 2, 3] + 3I_0^{[1,0]}[1, 3, 2] + 4m_s^2I_0^{[2,0]}[1, 2, 2] - 2I_0^{[2,0]}[1, 3, 3]))\},
\end{aligned}$$

$$\begin{aligned}
C_{G^2}^f = & 2/3m_s\{m_b^3[2m_s^4I_0[2, 1, 3] - (-1 + m_c^2)I_0[2, 1, 4] + m_s^2\{-3I_0[2, 1, 2] + 2(m_c^2I_0[2, 1, 3] \\
& + I_0^{[0,1]}[2, 1, 4])\}] + m_b^2m_c[-2m_s^4I_0[2, 1, 3] + (-1 + m_c^2)I_0[2, 1, 4] - 2I_0^{[0,1]}[2, 1, 4] \\
& + m_s^2(3I_0[2, 1, 2] - 2m_c^2I_0[2, 1, 3] + 4I_0^{[0,1]}[2, 1, 4] + 2I_0^{[1,0]}[2, 1, 4])] + m_b[-2m_s^6I_0[2, 1, 4] \\
& + I_0^{[0,1]}[2, 1, 3] - 2I_0^{[0,1]}[2, 1, 4] + I_0^{[0,2]}[2, 1, 2] + I_0^{[1,0]}[2, 1, 3] + m_c^2I_0^{[1,0]}[2, 1, 3] \\
& + m_s^4(5I_0[2, 1, 2] + 2I_0^{[1,0]}[2, 1, 3]) - I_0^{[1,1]}[2, 1, 4] + m_s^2(-4I_0[2, 1, 1] - 2I_0^{[0,1]}[2, 1, 3] \\
& + 6I_0^{[0,1]}[2, 1, 4] + m_c^2(3I_0[2, 1, 2] + 2I_0^{[0,1]}[2, 1, 4]) - 3I_0^{[1,0]}[2, 1, 4] + 2I_0^{[1,1]}[2, 1, 4])] \\
& + m_c[2I_0^{[0,1]}[2, 1, 3] + I_0^{[0,2]}[2, 1, 3] + m_s^4(-5I_0[2, 1, 2] + 2m_c^2I_0[2, 1, 3] - 6I_0^{[1,0]}[2, 1, 4]) \\
& - 3I_0^{[1,0]}[2, 1, 4] - I_0^{[2,0]}[2, 1, 4] + m_s^2(4I_0[2, 1, 1] - 3m_c^2I_0[2, 1, 2] - 4I_0^{[0,1]}[2, 1, 4] \\
& - 2I_0^{[0,2]}[2, 1, 2] + 9I_0^{[1,0]}[2, 1, 3] + 2I_0^{[2,0]}[2, 1, 4])]\}, \tag{74}
\end{aligned}$$

where $I_n[a, b, c]$ and $I_n^{[i,j]}[a, b, c]$ are defined as:

$$\begin{aligned}
I_0[a, b, c] &= \frac{(-1)^{a+b+c}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\mathbf{U}_0(a+b+c-4, 1-c-b), \\
I_1[a, b, c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{2-a-b}(M_2^2)^{3-a-c}\mathbf{U}_0(a+b+c-5, 1-c-b), \\
I_2[a, b, c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\Gamma(a)\Gamma(b)\Gamma(c)}(M_1^2)^{3-a-b}(M_2^2)^{2-a-c}\mathbf{U}_0(a+b+c-5, 1-c-b), \\
I_n^{[i,j]}[a, b, c] &= [M_1^2]^i[M_2^2]^j\frac{d^i}{d(M_1^2)^i}\frac{d^j}{d(M_2^2)^j}[M_1^2]^i[M_2^2]^jI_n[a, b, c]. \tag{75}
\end{aligned}$$

The function $\mathbf{U}_0(a, b)$ in the above formulas is given by

$$\mathbf{U}_0(a, b) = \int_0^1 dy (y + M_1^2 + M_2^2)^a y^b \exp[-\frac{B_{-1}}{y} - B_0 - B_1 y], \tag{76}$$

where

$$\begin{aligned}
B_{-1} &= \frac{m_b^2}{M_1^2} [M_1^2 + M_2^2], \\
B_0 &= \frac{1}{M_1^2 M_2^2} [M_1^2 m_c^2 + M_2^2 (m_c^2 + m_b^2)], \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2}.
\end{aligned} \tag{77}$$

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